# MATCH-UP 2020

1. If you were to write out every integer between 0 and 1,000,000 (this means you write 0, 1, 2, 3, 4, ..., 999,999, 1,000,000), how many times would you write the digit 2?

2. There is a four-digit even number $x$. The sum of the digits of $x$ is another number $y$. The last two digits of $y^2$ (itself a four-digit number) are the middle two digits of $x$. What is $x$?

3. Elle is buying fireworks for the 4th of July. She can buy packs of 7 from The Firework Shop and packs of 9 from The Firework Emporium. What is the most number of fireworks she cannot buy an exact amount of?
   For example, she can buy 25 fireworks from buying one pack from The Firework Shop and two packs from the Firework Emporium. However, she cannot buy exactly 10 fireworks.

4. Take any positive integer greater than 1. If it is even, divide it by 2. If it is odd, add 7 to it. Repeat the process. Many numbers will eventually reach 1 after enough repetitions, but not all. Find the $101^{st}$ such number that does NOT eventually reach 1.
   Example: The number 11 is odd, add 7 it to get 18. 18 is even, divide it by 2 to get 9. 9 is odd, add 7 to get 16. 16 is even, divide it by 2 to get 8. 8 is even, divide it by 2 to get 4. 4 is even, divide it by 2 to get 2. 2 is even, divide it by 2 to get 1.

5. An analog clock shows the correct time at 12:00 AM on October 1. At that point, it begins losing 1 minute and 7 seconds each hour. What date and time will it next show the correct time?

6. Pablos Restaurant has a catering menu of various items. One of the items on their catering menu is enchiladas. If a group of 11 people purchase the enchiladas and divide them evenly amongst everyone, there are 4 left over. If a group of 23 people purchase the enchiladas and divide them evenly amongst everyone, there are 7 left over. Assuming Pablo's only sells one size enchilada dish on their catering menu, what is the least number of enchiladas that would be in this dish?

7. A magic square is an $n \times n$ grid of numbers where every row, column, and main diagonal sums to the same value. For this question, consider a multiplicative magic square where every row, column, and main diagonal multiplies to the same non-zero value. Complete the following multiplicative magic square.

|     | 19,683 |       |
| --- | ------ | ----- |
|     |        |       |
| 729 |        | 6,561 |

8. The Pythagorean equation

$$a^2 + b^2 = c^2$$

has exactly one solution in consecutive positive integers $a$, $b$, $c$. (This solution is $3^2 + 4^2 = 5^2$.

Suppose we consider the "super-sassy Pythagorean equation"

$$a^2 + b^2 + c^2 + d^2 + e^2 = f^2 + g^2 + h^2 + i^2.$$

Is there a solution for consecutive positive integers $a < b < c < d < e < f < g < h < i$? If so, find such a solution. If not, answer "no solution".

9. Consider the quadratic equation $6x^2 + Bx + 6 = 0$. If this equation has a positive integer root and distinct roots, what is the maximum distance between $x$-intercepts of this equations associated parabola and what is the minimum distance between $x$-intercepts? Give both answers as an improper fraction (such as: $25/7$).

Fun fact! In the $15^{th}$ and $16^{th}$ centuries, mathematicians would challenge each other to public math duels. Very often, these duels would consist of solving equations (like this one) or factoring something that the other one could not.

10. The guard at a jail gives each of three prisoners a positive integer, but none of them know what the others have. He says he will give each one a clue, which they dont have to tell each other. If anyone can give the product of the three numbers, that prisoner can go free, but only that one. If a prisoner guesses wrong, they are sentenced to life in prison. The three prisoners debate for a moment, then decide to exchange clues.

- Prisoner 1's clue says their numbers sum to 71.
- Prisoner 2's clue says the highest and lowest numbers differ by 14.
- Prisoner 3's clue says that his number is smaller than everyone else's and it is a prime number.

All three prisoners look at each other, feeling confident. After deciding to count to three and all shout out the number at the same time, all three are released from prison. What answer did they all give?

11. The line $y = mx$ intersects the parabola $y = x^2 - \dfrac{1}{4}$ at two points, $A$ and $B$. Given that the length of the line segment $AB$ is 4, find $|m|$.

12. The streets of Sterling were frosted white by last night's snowstorm, so two neighboring families decided to take their children out for a walk through this beautiful winter wonderland. The Eatons and the Sanchezes bundled up their 7 children (4 of whom Becky, Irene, Nell, and Tina are girls and three of whom Frank, Leo, and Zachary are boys) and set out. The four parents (Mr. and Mrs. Eaton and Mr. and Mrs. Sanchez)

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walked down the recently shoveled sidewalk, each pushing a stroller containing at least one child, each of whom is either 1, 2, or 3 years old. The jaunt was a quick one, and before any of the children got too cold, everyone was back home enjoying some warm milk and cookies. From the information provided below, can you determine the child or children in the stroller pushed by each parent? (Note: the last names of the children are the same as their parents and each parent pushed a stroller containing only their own children.)

- Irene rode in the same stroller as her sister (who isn't Tina).
- There are more 3 year old children than 2 year old children.
- Irene Sanchez is younger than her brother.
- Mr. Eaton pushed a stroller containing at least one 2 year old child.
- Leo's mother pushed a stroller containing at least one 1 year old child.
- Neither Mr. Sanchez nor Mrs. Sanchez pushed the only stroller containing exactly three children.
- Nell and her twin sibling are both younger than Zachary but older than the only boy in Mr. Eatons stroller.